Dependence of the dynamical factor in nucleation rates on heat conduction and viscosity

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We reevaluate κ, a factor appearing in Langer and Turski's formulation of the nucleation rate for both nonrelativistic and relativistic systems with both heat conduction and viscosity. For the relativistic case, the expression for κ we present may be used in the calculation of quark drops forming in superheated baryon material with nonzero baryon number. We compare our result with numerical results in the literature and suggest that our treatment of the effects of viscosity leads to a reduction in the quark-gluon nucleation rate. [S1063-651X(96)08509-1]

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I. INTRODUCTION

The classical expression for the nucleation rate of drops or bubbles of one phase in another,

$$I=\frac{\kappa}{2\pi}\Omega_0\exp(-\Delta F/T), \tag{1}$$

given by Langer and Turski [1], has recently been applied to relativistic systems [2,3]. Here κ is related to the growth rate of bubbles or drops of radius R and critical radius R_0 through $dR/dt=\kappa(R-R_0)\equiv\kappa\tilde{R}$, Ω_0 is a statistical prefactor, and ΔF the difference in free energy of systems with, and without, a critical size drop present. Bubbles are formed by energy density fluctuations in the vapor. Those with radii R smaller than a critical size R_0 collapse, while those of the critical size begin to grow exponentially. In the relativistic generalization the form (1) is retained but ΔF, κ, and Ω_0 are obtained using the appropriate equation of state for the relativistic material and the relativistic hydrodynamical equations. In this paper we will follow Refs. [2,3] in assuming the form (1), based on the nonrelativistic Langer formalism, for the relativistic nucleation rate and will focus primarily on the derivation of the factor κ. Expressions for Ω_0 and ΔF are given in [2] and the third reference of [3]. We wish to point out that we have no proof that the Langer nucleation rate formalism can be extended to relativistic systems as has been done in [2,3].

In particular, (1) has been applied to the calculation of the relativistic nucleation rate of hadron bubbles in quark vapor [2,3] and to the similar problem of quark drop formation in superheated baryon matter. Csernai and Kapusta [2] in applying the Langer-Turski nucleation rate, (1), to the formation of a hadron bubble in quark vapor, have suggested that

$$\kappa=\frac{4\sigma}{(\Delta w)^2R_0^3}\left(\frac{4}{3}\eta+\zeta\right), \tag{2}$$

where η and ζ are the shear and bulk viscosity coefficients in the vapor region, respectively, σ is the surface tension, and Δw the difference in enthalpy per volume between the two phases. The result (2) is proposed for a relativistic system with no heat conduction and so applies to a system of zero baryon number.

Venugopalan and Vischer [4] have considered systems with both heat conduction and viscosity and suggest that in such cases

$$\kappa=\frac{2\sigma}{(\Delta w)^2R_0^3}\left[\lambda T+2\left(\frac{4}{3}\eta+\zeta\right)\right], \tag{3}$$

where λ is the coefficient of thermal conductivity. They claim that (3) is valid in both the nonrelativistic and the relativistic cases (provided there is a net baryon number). The expression (3) has been used in recent calculations of the nucleation rate of quark drops in superheated baryon material [3]. In the case of no net baryon number the authors of [4] suggest setting the λ term equal to zero and thus obtain (2). In the limit of no viscosity they obtain the nonrelativistic result of Langer and Turski [1]. The work of [4] combines techniques and ideas from both the nonrelativistic work of [5] and the relativistic work of [2] for systems of zero baryon number.

While we have recently calculated κ in the case of zero baryon number (and no heat conduction) in [6] and found a result differing from that of [2], the objective of the present work is to reexamine the derivation of κ for systems with nonzero baryon number which allow for heat conduction as a mechanism for energy removal in phase transitions. In particular, we focus on two main points. First we are interested in the form of viscosity terms in κ in relationship to those of heat conduction. A numerical evaluation of (3), for the case of quark-gluon drops forming in superheated baryon material [3], shows that the viscous term dominates the heat conduction term. This implies that viscosity is important for nucleation. Such a result is not only significant for relativistic systems but is also relevant to the nonrelativistic case examined by Langer and Turski because (3) is suggested to hold for both [4]. Langer and Turski have focused on the effects of heat conduction and ignored viscosity in [1]. If viscosity is important then it should be incorporated into the nonrelativistic expression for κ. If on the other hand (3) must be modified and a new expression shows that the effects of viscosity are small then the Langer-Turski result [1] is a good approximation for viscous systems. In addition if the effects of viscosity are small the nucleation rate for quark-gluon drops in baryon material would be reduced from the values given in the third reference of [3]. We specifically investigate these

possibilities in this paper. As a second goal we wish to determine the differences in κ, if any, arising from heat conduction for relativistic versus nonrelativistic systems. We consider both nonrelativistic and relativistic systems in our investigation of the validity of an expression such as (3). This allows us to obtain an expression for κ for relativistic systems in the presence of viscosity which we believe to be more accurate than (3).

To introduce our approach, we first calculate κ for a nonrelativistic drop, forming in a vapor which is a perfect heat conductor, both in the presence and absence of viscosity. We then examine the arguments used in [4] and suggest that a different way of finding κ is necessary. We propose a method similar to that presented in [6] which leads to a result differing from (3). We explicitly solve for κ in the presence of finite heat conduction and viscosity for a nonrelativistic problem to obtain an expression which, in the absence of viscosity, yields the result of [1]. Next we examine how the treatment of heat conduction differs in relativistic and nonrelativistic problems. We present a result for the former in the absence of viscosity before treating a relativistic material which requires the treatment of both heat conduction and viscosity to obtain an expression for κ. Finally we present some numerical results for superheated baryon material undergoing a process of quark-gluon drop nucleation to determine the physical importance of the viscosity terms and the relativistic modifications to heat conduction. We determine if the numerical dominance of the viscosity terms in (3) applies for our expression for κ and investigate how any change in (3) affects the overall nucleation rate. To conclude we examine the scaling properties of (3) and the expression for κ which we derive, following the treatment of scaling given by Kawasaki [7].

II. VISCOSITY EFFECTS FOR A PERFECT HEAT CONDUCTOR

It is possible to solve for κ for a vapor which is a perfect heat conductor both in the presence and absence of viscosity. Such an example is not very physical because it assumes heat can be removed quickly enough to keep the temperature constant. This example, however, simplifies the calculation because variables, such as pressure, depend on the density and not on both density and temperature. The example of the perfect heat conductor without viscosity was considered in [1]. We develop this example to illustrate the modifications introduced by viscosity, and to introduce both the method and also some general features and results which arise in the more general case where both density and temperature vary.

To begin we consider the case of a nonviscous vapor using the method of [6] based on the solution of the transport equations in the vapor and liquid regions together with boundary conditions which link one to the other. The method differs from the approach of [1], which uses differential equations in the narrow interface region where the density changes from that of liquid to that of vapor. The result of [1] is obtained in the limit of small κ. After treating the case of a nonviscous vapor we later will turn to modifications due to viscosity.

The basic transport equations which apply to the nonrelativistic case are

$$\partial_t \tilde{n} = -\vec{\nabla}\cdot(n\vec{u}),$$

$$n\partial_t \vec{u} = -\vec{\nabla}\tilde{p}. \tag{4}$$

We set the mass per particle equal to 1 in this section for simplicity. Equations (4) result from a linearization of the standard number density continuity and Euler equations [6] about number density $n+\tilde{n}$ and fluid velocity $\vec{0}+\vec{u}$. We consider flow in the radial direction only. We apply Eqs. (4) to the vapor region and ignore the solution in the liquid region as the fluid velocity is small there compared with that in the vapor (see [1]). We relate $\tilde{p}_v$ to $\tilde{n}_v$ as done in [1]:

$$\tilde{p}_v = \frac{\partial p}{\partial n}\tilde{n}_v = c^2\tilde{n}_v . \tag{5}$$

Here $\tilde{n}_v$ is the variation of density, about a stationary system, of a drop in unstable equilibrium with a vapor, and c is the speed of sound in the vapor. The subscripts v and l are to be used to denote the vapor and liquid regions, respectively. The boundary conditions at the surface of the liquid drop are

$$p_l - p_v = \frac{2\sigma}{R},$$

$$\Delta n \frac{dR}{dt} = -n_v u_v , \tag{6}$$

where

$$\Delta n = n_l - n_v . \tag{7}$$

We have specifically set u_l to zero.

The two equations of (4) can be combined with (5) to give the wave equation

$$\partial_t^2 \tilde{n} = c^2 \vec{\nabla}^2 \tilde{n}. \tag{8}$$

A solution of this equation has the form

$$\tilde{n}_v \propto \exp(\kappa t)\frac{\exp[-q(r-R_0)]}{r}, \tag{9}$$

where

$$\kappa^2 = c^2 q^2. \tag{10}$$

In addition the first equation of (4), with $\tilde{n}_v$ given by (9), yields

$$u = \frac{\kappa}{n_v q^2}\left(q+\frac{1}{r}\right)\tilde{n}_v . \tag{11}$$

To obtain κ one must make use of the boundary conditions. To do so one may first differentiate (5) with respect to time. Next a relationship for $\tilde{p}$ is obtained from the condition [1,5]

$$p_v = \frac{n_v 2\sigma}{R_0 \Delta n} \tag{12}$$

by varying R about R_0, namely,

$$\tilde{p}_v = -\frac{n_v 2\sigma \tilde{R}}{R_0^2 \Delta n}. \tag{13}$$

Equation (13) can be differentiated with respect to time and (6) used to replace $\dot{R}$ in terms of u. Finally solutions (9) and (11) may be used to obtain

$$\frac{n_v 2\sigma}{(\Delta n)^2 R_0^2}\left(q+\frac{1}{R_0}\right)=c^2 q^2, \tag{14}$$

where q can be written in terms of κ using (10). In the limit of small κ we obtain the result of [1] for nonviscous systems:

$$\kappa = \sqrt{\frac{2\sigma n_v}{R_0^3 (\Delta n)^2}} \tag{15}$$

by neglecting q relative to $1/R_0$ in (14).

The inclusion of viscosity affects both Eqs. (4) and the boundary conditions (6). Considered to first order in u the second equation of (4) becomes

$$n\partial_t \vec{u} = -\vec{\nabla}\frac{\partial p}{\partial n}\tilde{n} + \vec{\nabla}\left(\frac{4}{3}\eta+\zeta\right)\vec{\nabla}\cdot\vec{u} \tag{16}$$

while the first equation of (4) is unchanged. The first condition of (6) is, however, modified when viscosity is included while the second condition of (6) remains unchanged to first order in u. The first of Eqs. (6) becomes

$$\tilde{p}_l + b_v - \tilde{p}_v - b_v = -\frac{2\sigma\tilde{R}}{R_0^2}, \tag{17}$$

where

$$b_i = -\left(\frac{4}{3}\eta_i+\zeta_i\right)\vec{\nabla}\cdot(\vec{u}_i-\dot{\vec{R}}) + \frac{4\eta_i(u_i-\dot{R})}{R_0}. \tag{18}$$

The terms b_i, which arise in Eq. (17), are a consequence of the continuity of the energy-momentum tensor in a frame moving with the drop surface [9,6] (see Appendix B). The specific factor

$$(u_i-\dot{R}) \tag{19}$$

present in (18) represents the fluid velocity in a frame moving with the liquid-vapor interface.

The transport equations including viscosity effects can be combined to obtain the following replacement for (8):

$$\partial_t^2 \tilde{n} = c^2 \vec{\nabla}^2 \tilde{n} + \frac{1}{n_v}\left(\frac{4}{3}\eta+\zeta\right)\partial_t \vec{\nabla}^2 \tilde{n}. \tag{20}$$

A solution of this equation is still of the form (9) but with the dispersion relation between κ and q given by

$$\kappa^2 = \left(c^2 + \frac{\left(\frac{4}{3}\eta_v+\zeta_v\right)}{n_v}\kappa\right) q^2. \tag{21}$$

The boundary condition (17) must next be manipulated to obtain a new expression for $\tilde{p}_v$ analogous to (13) so that the same method of obtaining κ for the nonviscous case can be applied here. To find such an expression for $\tilde{p}_v$, the terms involving u in (18) can be reexpressed in terms of density using the transport equations. For example, a $\vec{\nabla}\cdot\vec{u}$ term is replaced with a term involving $\kappa\tilde{n}$ using the first equation of (4), $\vec{\nabla}\cdot\dot{\vec{R}}$ is replaced with $(2/R_0)\dot{R}$. Finally (9) and (11) are used to express u in terms of $\dot{R}=\kappa\tilde{R}$. The result is

$$\tilde{p}_l + b_l(\tilde{n}_v,\tilde{R}) - \tilde{p}_v - b_v(\tilde{n}_v,\tilde{R}) = -\frac{2\sigma\tilde{R}}{R_0^2}. \tag{22}$$

A problem arises in (22) since here one has two unknowns $\tilde{p}_l$ and $\tilde{p}_v$. Such a problem does not occur in the relativistic zero baryon number problem of [6] as pressure and energy density are given as functions of temperature alone. (For details see Ref. [6].) In the absence of viscosity (13) and

$$\tilde{p}_l = -\frac{2\sigma\tilde{R}}{R_0^2}\frac{n_l}{\Delta n} \tag{23}$$

apply at the drop surface. In the viscous case the conditions (13) and (23) cannot both hold. We choose to retain (23), as the velocity and thus viscous terms are neglected inside the drop, to obtain a particular solution for $\tilde{p}_l$. Using (5) to express $\tilde{n}_v$ in terms of $\tilde{p}_v$ and grouping $\tilde{R}$ terms one may obtain

$$\tilde{p}_v = -\frac{\tilde{R}B_1}{B_2}, \tag{24}$$

where

$$B_1(\kappa) = \frac{2\sigma n_v}{R_0^2\Delta n} - \left(\frac{4}{3}\eta_l+\zeta_l-\frac{4}{3}\eta_v-\zeta_v\right)\frac{2\kappa}{R_0} + \kappa\left(\frac{4\eta_l}{R_0}-\frac{4\eta_v n_l}{n_v R_0}\right) \tag{25}$$

and

$$B_2(\kappa) = c^2 \Big/ \left[c^2 + \left(\frac{4}{3}\eta_v+\zeta_v\right)\frac{\kappa}{n_v}\right]. \tag{26}$$

The method employed in the nonviscous case can now be applied and one obtains, in the limit of small κ,

$$\kappa = \sqrt{\frac{B_1(\kappa_0)}{R_0\Delta n}}. \tag{27}$$

Here κ_0 is the result in the absence of viscosity (15). Although the condition of perfect heat conduction assumed in this section may be unrealistic, the result for κ illustrates the general approach we use. It shows that there are two modifications due to viscosity, one in the dispersion relation between κ and q, the other in the expression for $\tilde{p}_v$. Although the only independent variable for the perfect heat conduction case is density these two types of modifications are also present in the more general case where temperature and density change. It also provides qualitative information on the role of viscosity in the growing droplet problem. The modification to c (21) introduces a small change in κ, when κ is

small. The viscosity terms in B_1 and B_2, arising from the modified boundary conditions, also introduce a small correction under the same circumstance because they are multiplied by κ itself.

III. DISCUSSION OF METHODS OF REF. [4]

Reference [4] has presented an expression for κ, for the case of a drop, growing in a vapor with both finite heat conduction and viscosity, namely (3). The approach of Ref. [4] is based on the Kotchine conditions and closely follows the work of [5] which, however, ignores viscosity. We examine the method of [4] before considering an alternative method of calculating κ in the next section. We will find there an expression for κ differing from (3).

The three key equations of Ref. [4] are

$$[nu_R]=[n]dR/dt, \tag{28}$$

$$\Delta w dR/dT=-\lambda dT/dr-(4/3\eta+\zeta)u_R du_R/dr, \tag{29}$$

and

$$\Delta w {u_R}^2=2\sigma(1/R_0-1/R). \tag{30}$$

Here Δw represents the change in enthalpy density and the square brackets in (28) denote the difference of the quantity inside the brackets evaluated inside and outside the drop surface at R. The first of these equations, (28), represents the conservation of matter for the growing drop. The second gives the energy flux and describes how the released latent heat is removed by heat conduction and viscosity. The third, Eq. (30), is referred to as a momentum flux equation.

The first equation (28) with $dR/dt=\kappa(R-R_0)$, and $u_{Rl}=0$ so that $[nu_R]=n_v u_{Rv}$, implies that $u_{Rv}\propto(R-R_0)$. Here u_{Rv} is the vapor fluid velocity evaluated at the drop surface. $(R-R_0)\propto\exp(\kappa t)$ and so gives the time dependence of u. Equation (30), however, suggests that $u_2\propto(R-R_0)$ in contrast with the result of (28). This suggests a different time dependence for u. The constrast in the time dependences of u in (30) and (28) is a problem as u is understood to be separable in time and space coordinates. The separability of u is not so clear in the derivation of [4] but is very evident in the differential equation approaches of [2,1].

Equation (30), and not (28), is used in the calculation of Ref. [4] to replace u_R in terms of $(R-R_0)$ which may then be cancelled from both sides of (29) to obtain a result for κ. [Note in Ref. [4] it is shown that $dT/dr\propto(R-R_0)$.] This leads to the conclusion that the heat conduction and viscosity terms are of equal importance. We suggest from Eq. (28) that $u_R\propto(R-R_0)$. This implies that the viscosity term is of higher order and has a different time dependence than the other two terms in (29). (One may note that in [1] all second order terms in u are dropped when solving the transport equations.)

We now suggest that the inconsistency in the time dependency of u can be avoided if one rejects (30). The approach of [1] follows this route. We focus on the calculation of κ for a perfect heat conductor in Ref. [1] to demonstrate that (30) is not used. (A minor error in [1], corrected in [5], does not involve the part of the calculation focusing on the perfect heat conductor.) Langer and Turski, in [1], solve the two differential equations

$$\frac{\partial n}{\partial t}=-\vec{\nabla}\cdot(n\vec{u}) \tag{31}$$

and

$$\frac{\partial \vec{u}}{\partial t}=-\frac{\vec{\nabla}p}{mn}-\frac{1}{2}\vec{\nabla}u^2 \tag{32}$$

using $n=n(\text{equilibrium})+\tilde{n}(r)e^{\kappa t}$, $u\propto e^{\kappa t}$, and $p=c^2\tilde{n}$. The velocity is assumed to be purely radial in (32). The general form of the second term on the right-hand side of (32) is $-\vec{u}\cdot\vec{\nabla}\vec{u}$. The first equation (31) is identical to the Kotchine result (28). The second, the Euler equation, is used to replace u in (31). Langer and Turski drop the term of order u^2. Equation (32) is used to replace (30) in its role in finding a way to replace u but in this case it is linear rather than quadratic in u. In the presence of viscosity a modified version of the Euler equation, i.e., the Navier-Stokes equation, is needed. If the vapor is not a perfect heat conductor then an additional equation describing the change in temperature with time and the rate of conduction is needed. In such a case one would solve a total of three transport equations as shown in [2]. The energy rate equation (29) appears to incorporate the information of these three equations but would still need to be solved together with (28) and a Navier-Stokes equation.

IV. NONRELATIVISTIC CASE: HEAT CONDUCTION AND VISCOSITY

We wish to present a method to solve for κ which combines the transport differential equations with the Kotchine boundary equations. The method, which differs from that of [1,5], leads to a general solution for κ which agrees with the small κ limit presented in [1,5]. For clarity we first examine the case in which viscosity is absent. Then we introduce modifications due to viscous effects. The two key transport equations for the case of nonperfect heat conduction (finite conductivity λ) are

$$\kappa\theta=-\frac{T}{nc_v}\frac{dp}{dT}(\vec{\nabla}\cdot\vec{u})+\frac{\lambda}{nc_v}\vec{\nabla}^2\theta \tag{33}$$

and

$$\kappa^2\tilde{n}=\frac{1}{m}\vec{\nabla}\cdot n\vec{\nabla}(-K\vec{\nabla}^2+f'')\tilde{n}+\frac{1}{m}\vec{\nabla}^2\frac{dp}{dT}\theta. \tag{34}$$

Here c_v is heat capacity, p pressure, and K a constant related to surface tension. The term $\theta(r)$ is the change in temperature from the equilibrium value due to the release of latent heat energy. The pressure is related to the Helmholtz free energy density f through

$$p=n\frac{\partial f}{\partial n}-f. \tag{35}$$

Equation (34) is the Navier-Stokes equation combined with a density conservation equation. We take the solution of Eqs. (33) and (34), in the vapor region, from [1], i.e.,

$$r\tilde{n}=A_1\exp[-q_1(r-R_0)]+A_2\exp[-q_2(r-R_0)],$$

$$r\theta=\left(\frac{A_1}{1-b}\exp[-q_1(r-R_0)]+A_2\exp[-q_2(r-R_0)]\right)\frac{T(\partial p_v/\partial T)}{c_v n_v^2}, \quad (36)$$

where

$$b\equiv 1+\left[T\left(\frac{dp_v}{dT}\right)^2\Big/\, c_v n_v^3\left(\frac{\partial^2 f}{\partial n_v^2}\right)\right]. \quad (37)$$

Explicit values for q_1 and q_2 are obtained by solving

$$\det\begin{vmatrix} \kappa^2-n_v q^2\dfrac{\partial^2 f}{\partial n^2 m} & -q^2(\partial p_v/\partial T)/m \\ \dfrac{-T(\partial p_v/\partial T)}{n_v^2 c_v}\kappa & \kappa-\dfrac{\lambda q^2}{n_v c_v}\end{vmatrix}=0.$$

In the limit of small κ

$$q_1^2=c_v n_v \kappa b/\lambda,$$

$$q_2^2=\frac{m\kappa^2}{n_v b(\partial^2 f/\partial n_v^2)}. \quad (38)$$

Here

$$c^2\equiv n_v\frac{\partial^2 f}{\partial n_v^2}, \quad (39)$$

where c^2 is the speed of sound in the vapor. The above expressions for q_1 and q_2 have been given in [1].

Next we obtain an expression for the ratio of A_2/A_1, A_1, and A_2 being the coefficients in (36). To do so we make use of the boundary conditions (6) together with the condition from [5],

$$n_l l\frac{dR}{dt}=-\lambda\frac{dT}{dr}, \quad (40)$$

where l is the latent heat. Condition (40) implies that the latent heat energy released in a phase transition is carried away by pure heat conduction. This condition is consistent with the work of [1] as is shown in Appendix A. One may then obtain

$$\frac{l n_l n_v}{\Delta n}u=-\lambda\frac{dT}{dr}, \quad (41)$$

using (40) and the boundary conditions (6), which together with

$$u_v=\frac{\kappa}{n_v}\left[\frac{A_1}{q_1^2}\left(\frac{1}{R_0^2}+\frac{q_1}{R_0}\right)+\frac{A_2}{q_2^2}\left(\frac{1}{R_0^2}+\frac{q_2}{R_0}\right)\right], \quad (42)$$

the velocity evaluated at the boundary $r=R_0$, give a relationship between A_1 and A_2. Specifically we find

$$\frac{A_2}{A_1}\approx m\kappa\lambda\left(\frac{T(\partial p_v/\partial T)\Delta n}{c_v n_v^2(1-b)n_l l b c^2}+\frac{1}{b^2 c^2 n_v c_v}\right). \quad (43)$$

A number of approximations are made to obtain (43). First, the q_i/R_0 terms in (42) have been dropped relative to $1/R_0^2$. Second, only the A_1 term in (36) is retained because it is assumed, and later confirmed in (43), that $A_1 \gg A_2$. Equation (43) is consistent with the work of [1]. In obtaining (43) we have retained both the $A_1 q_1^{-2}$ and $A_2 q_2^{-2}$ terms. In [1] the former is dropped as an approximation. This is consistent with dropping the second term on the right-hand side of (43). We also adopt this approximation in the calculation of κ to be consistent with [1]. In Appendix A we calculate A_2/A_1 using the formalism of [1] to show that it is identical to the first term on the right-hand side of (43).

Finally one can form an expression for κ using

$$\tilde{p}=c^2\tilde{n}+\frac{dp}{dT}\theta, \quad (44)$$

with

$$\tilde{p}=0,$$

$$\theta=\frac{2\sigma T\tilde{R}}{l n_l R_0^2} \quad (45)$$

given in [5]. The expression for θ, given by (45), holds at $R=R_0$. In general θ is a function of r. We discuss the origins of both conditions in (45) in some detail so that it is clear how they may be modified when viscosity is introduced. It is interesting to note that in the absence of a temperature change, θ, one would have $\tilde{p}_v \propto -2\sigma/R_0^2$. Physically this means that the vapor pressure is being lowered as the drop grows. The drop pressure is also being lowered. This is reasonable because in the coexistence equilibrium situation the pressure of both a large amount of vapor and liquid is lower than both the liquid and vapor pressures in the supersaturated region for T constant. The presence of θ due to heat being released leads to an increase in the vapor pressure. The approximation in [1] is that $\tilde{p}_v$ not change while both temperature and density do change. This approximation is consistent with κ being small. To examine the approximation in more detail one evaluates $\tilde{n}$ and θ (36) at the surface to find

$$\frac{\tilde{n}}{\theta}=\frac{A_1+A_2}{[A_1/(1-b)+A_2]}\frac{c_v n_v^2}{T\partial p/\partial T}. \quad (46)$$

Using the condition (43) and assuming that κ is small allows one to neglect A_2 relative to A_1. Using the expression for b, (37), we then find

$$\frac{\tilde{n}}{\theta}=-\frac{c^2}{dp/dT}, \quad (47)$$

which is just the result obtained using (44) and $\tilde{p}_v=0$.

The second condition of (45) can be obtained by keeping the temperature change and the chemical potential across the liquid-vapor boundary the same and using the thermodynamic result [9]:

$$d\left(\frac{\mu}{T}\right)=-\frac{w}{nT^2}dT+\frac{dp}{nT}. \tag{48}$$

This leads to

$$\frac{dp_v}{n_v}-\frac{dp_l}{n_l}=\frac{dT}{T}\left(\frac{w_l}{n_l}-\frac{w_v}{n_v}\right). \tag{49}$$

Here dp and dT are understood to be changes in pressure and temperature between a situation of a drop in unstable equilibrium with a surrounding vapor and a perturbed, growing drop, i.e.,

$$dp_v\equiv\tilde{p}_v,$$

$$dT\equiv\theta. \tag{50}$$

Equation (49) is equivalent to the expression

$$\frac{P_l}{n_l}-\frac{P_v}{n_v}=\frac{dT}{T}\left(\frac{w_l}{n_l}-\frac{w_v}{n_v}\right), \tag{51}$$

where P_v in (51) is the deviation from the pressure of a large amount of liquid in equilibrium with a vapor with negligible surface effects present. For a drop in unstable equilibrium

$$\overline{P_i}=\frac{n_i}{\Delta n}\frac{2\sigma}{R_0}, \tag{52}$$

hence

$$\frac{\overline{P_v}}{n_v}-\frac{\overline{P_l}}{n_l}=0 \tag{53}$$

and so dp in (49) can be replaced with $P=\overline{P}+dp$, which is consistent with the work of [5]. Using

$$\mu_v=\mu_l \tag{54}$$

and the relationship between Gibb's free energy and the chemical potential

$$\mu_v N_v=G_v, \tag{55}$$

where N_v is the number of vapor particles, one can show that

$$\frac{w_v}{n_v}-\frac{w_l}{n_l}=l, \tag{56}$$

l being the latent heat per particle. Thus one is led to an expression

$$\frac{\tilde{p}_l}{n_l}-\frac{\tilde{p}_v}{n_v}=-\frac{l\theta}{T}. \tag{57}$$

This expression may be used to solve for θ at the boundary as has been shown in detail in [5].

To obtain an explicit expression for κ one differentiates (44) with respect to time with values for $\tilde{p}_v$ and θ given by (45). Next one replaces $\dot{R}$, arising from $\dot{\theta}$, in terms of u_v using (6) to obtain

$$c^2\kappa\tilde{n}=\partial_T p\frac{2\sigma T}{n_l R_0^2}\frac{n_v u_v}{\Delta n}. \tag{58}$$

$\tilde{n}$ and u_v can be evaluated using (36), (42), and (43). Dropping terms with $A_1 q_1^{-2}$ in comparison with $A_2 q_2^{-2}$ as well as terms q relative to $1/R_0$ in u (42) leads to an expression for κ. We also neglect A_2/A_1 relative to 1. Under the assumption that κ is small ($\ll 1$):

$$\kappa=\frac{2\sigma T\lambda}{(n_l l)^2 R_0^3}, \tag{59}$$

which agrees with the corrected results of [1,5].

In the presence of viscosity, with viscosity coefficients η and ζ, modifications to first order in u lead to the replacement of (34) with

$$\kappa^2\tilde{n}=\frac{1}{m}\vec{\nabla}\cdot n\vec{\nabla}(-K\vec{\nabla}^2+f'')\nu+\frac{1}{m}\vec{\nabla}^2\frac{dP}{dT}\theta$$
$$+\vec{\nabla}\cdot n\vec{\nabla}\left(\frac{4}{3}\eta+\zeta\right)\kappa\tilde{n}\frac{1}{n}. \tag{60}$$

Equation (33) remains unchanged to first order in u. It is possible to incorporate the first and third terms on the right-hand side of (60) and replace c^2 with

$$c^2+\left(\frac{4}{3}\eta_v+\zeta_v\right)\frac{\kappa}{n_v m} \tag{61}$$

in (38) and (43). The boundary conditions with viscosity become

$$n_l u_l-n_v u_v=\Delta n\frac{dR}{dt},$$

$$p_l+n_l(u_l-\dot{R})^2+b_l-p_v-n_v(u_v-\dot{R})^2-b_v=\frac{2\sigma}{R_0},$$

$$n_l l\frac{dR}{dT}=-\lambda\frac{dT}{dr}-2\eta u\partial_r u-\left(\zeta-\frac{2}{3}\eta\right)u\vec{\nabla}\cdot\vec{u}. \tag{62}$$

The conditions (62) are a consequence of the continuity of the energy-momentum tensor [9,6]. Here b_i is given by (18). We, however, drop the viscosity terms of second order in u. These modified boundary conditions lead to forms of $\tilde{p}_v$ and θ different than those used in the nonviscous case (45). We now determine these new expressions. The condition, $\tilde{p}_v=0$, is altered as the ratio of $\tilde{n}$ to θ is changed from (47) to

$$\frac{\tilde{n}_v}{\theta}=(1-b')\left(\frac{T\partial p/\partial T}{c_v n_v^2}\right)^{-1}, \tag{63}$$

where

$$1-b'=-\frac{T(\partial p/\partial T)^2}{mc_v n_v^2 c^2 F},$$

$$F=\left[1+\left(\frac{4}{3}\eta_v+\zeta_v\right)\frac{\kappa}{n_v mc^2}\right]. \quad (64)$$

Here we have made use of (36) with $A_1 \gg A_2$ and the modification (61). Thus (44) together with (63) give

$$\tilde{p}_v=\theta(\partial p/\partial T)\left(1-\frac{1}{F}\right). \quad (65)$$

To find θ, expressions involving $\vec{\nabla}\cdot u$ in the second equation of (62) are reexpressed in terms of n_v and expressions involving $\vec{\nabla}\cdot\dot{R}$, $\dot{R}$, and u alone are reexpressed in terms of $\tilde{R}$ in

$$\tilde{p}_l(\tilde{n}_l,\theta)+b_l(\tilde{n}_v)-\tilde{p}_v(\tilde{n}_v,\theta)-b_v(\tilde{n}_v)=-\frac{2\sigma\tilde{R}}{R_0^2}. \quad (66)$$

First (66) is solved for $\tilde{p}_l$ in terms of $\tilde{n}_v$, $\tilde{R}$, and θ. Then (63) is used to replace $\tilde{n}_v$ in terms of $\tilde{\theta}$. Finally $\tilde{p}_l$, written as a function of θ and $\tilde{R}$, is inserted into (57) and a relationship between θ and $\tilde{R}$ obtained:

$$\theta=\frac{D_1(\kappa)\tilde{R}}{D_2(\kappa)}, \quad (67)$$

where

$$D_1(\kappa)=\frac{-2\sigma}{R_0^2}+\frac{2\kappa}{R_0}\left(-\frac{4}{3}\eta_l-\zeta_l+\frac{4}{3}\eta_v+\zeta_v\right)+\kappa\left(\frac{4\eta_l}{R_0}-\frac{4n_l\eta_v}{n_v R_0}\right) \quad (68)$$

and

$$D_2(\kappa)=\frac{-ln_l}{T}+\frac{\Delta n}{n_v}\frac{F-1}{F}\frac{\partial p}{\partial T}+\frac{n_l}{mF}\frac{dp}{c^2 dT}\times\left(\frac{4}{3}\eta_v+\zeta_v\right)\frac{\kappa}{n_v}. \quad (69)$$

This is to be compared with (45), for the nonviscous case. Expression (67), which includes viscosity induced modifications, also contains κ. To obtain an expression for κ one may differentiate (44) with respect to time using (65):

$$\dot{\theta}\frac{\partial p/\partial T}{F}=c^2\dot{\tilde{n}}_v. \quad (70)$$

Then one replaces the $\dot{R}$ in $\dot{\theta}$ in terms of u_v. Finally, one uses the forms (36) and (42) together with (43) to evaluate $\tilde{n}_v$ and u_v. Again only the A_1 term in $\tilde{n}_v$ and the $A_2 q_2^{-2}$ in u_v are retained. For the viscous case q_2^2 is replaced with $q_2^2(b/b'F)$ in u_v and b with b', given by (63), and c^2 with Fc^2 in A_2/A_1. One finally obtains

$$\kappa=\frac{D_1(\kappa_0)\lambda}{R_0 l n_l D_2(\kappa_0)}. \quad (71)$$

This is understood to hold in the limit of small κ. As a first approximation we have replaced κ in (67) with the result for κ_0, the value in the absence of viscosity (59).

V. RELATIVISTIC CASE: HEAT CONDUCTION BUT NO VISCOSITY

The energy flux associated with heat conduction is not the same in the relativistic and nonrelativistic cases. In the former case one has [9]

$$-\lambda\left(\vec{\nabla}T-\frac{T}{w}\vec{\nabla}p\right) \quad (72)$$

while in the latter

$$-\lambda\vec{\nabla}T. \quad (73)$$

The work of [4] suggests the same form for both, i.e., (29). We examine the relativistic case in detail here to see if the expression for κ is altered from the nonrelativistic result. For simplicity we ignore viscosity in this section. One may note that in the nonrelativistic limit $w\to e\gg p$ and the two expressions, (72) and (73), for heat conduction become the same. In order to determine the effects of heat conduction on κ for the relativistic situation we solve the following transport equations

$$\partial_t\tilde{e}=-w\vec{\nabla}\cdot\vec{u},$$

$$w\partial_t\vec{u}=-\vec{\nabla}\tilde{p},$$

$$-\partial_t\tilde{n}-\vec{\nabla}\cdot(n\vec{u})=-\lambda\left(\frac{nT}{w}\right)^2\left(-\frac{w}{nT^2}\partial_t^2 T+\frac{1}{nT}\partial_t^2 p\right)-\lambda\frac{n^2T^2}{w^2}\left(\frac{-w}{nT^2}\vec{\nabla}^2T+\frac{1}{nT}\vec{\nabla}^2p\right). \quad (74)$$

Here $\tilde{e}$ and $\tilde{p}$ are variations in the energy density and pressure. Terms of order u^2 and higher have been dropped in the approximation we use (see [6], and references therein) and the speed of light set equal to one. In addition the solutions in the interior are considered to be very small and so neglected, as an approximation (see Ref. [6]). It is important to note that u is the velocity of energy flow and not that of particle flow for a relativistic problem with dissipative effects, i.e., heat conduction here. (In the nonrelativistic limit the velocity u can be nonzero even in the case that the particle flow velocity is zero.) The following boundary conditions are assumed:

$$p_l-p_v=\frac{2\sigma}{R},$$

$$\mu_l=\mu_v,$$

$$w_v u_v=\Delta w\dot{R},$$

$$\Delta w=w_v-w_l. \quad (75)$$

Note (48) and other thermodynamic relations hold both in relativistic and nonrelativistic problems because e, p, and n, the energy density, pressure, and number density, respectively, are evaluated in a frame at rest with respect to the fluid in the formalism of relativistic fluid mechanics [9].

One must find a relationship analogous to (40) for the relativistic case in order to obtain an expression for κ. First, we use an additional boundary condition involving the continuity of the particle number density n:

$$n_l(u_l-\dot{R})+\nu_l=n_v(u_v-\dot{R})+\nu_v\,,$$

$$-\Delta n\dot{R}\approx n_v u+\nu_v\,. \tag{76}$$

Here $u=u_v$. Note $u-\dot{R}$ is the velocity of energy flow relative to the velocity of the drop surface using a Galilean transformation because the velocities u and $\dot{R}$ are understood to be small in the approximation used. We have again ignored the energy flow in the drop interior. An expression for ν, where $\vec{\nu}=\nu\vec{r}/r$, is given in [9]:

$$\nu_i=-\lambda\left(\frac{nT}{w}\right)^2\left[\frac{\partial}{\partial x^i}\left(\frac{\mu}{T}\right)+u_i u^k\frac{\partial}{\partial x^k}\left(\frac{\mu}{T}\right)\right]. \tag{77}$$

Here μ is the chemical potential. The second term on the right-hand side of (77) is dropped to be consistent with [9] and (48) used to obtain an expression for μ/T. One may now solve (76) for u and substitute into the third condition of (75). We obtain for the vapor region

$$-n_l l\ddot{\vec{R}}=\lambda\vec{\nabla}T-\frac{T}{w_v}\lambda\vec{\nabla}\tilde{p}, \tag{78}$$

where an expression for l is obtained from (56). One may note that dropping the second term on the right-hand side of (78), which disappears in the nonrelativistic limit, yields (40). One may use the second equation of (74) together with the relations $\partial_t u=\kappa u$ and $\dot{R}=\kappa\tilde{R}$ to solve for $\vec{\nabla}\tilde{p}$ in terms of $\tilde{R}$. Given that $\dot{R}=\kappa\tilde{R}$ all that remains is to find an expression for $\vec{\nabla}\theta$ in terms of $\tilde{R}$. Then (78) yields an expression for κ. One may note that this is a different approach to finding κ than that used in earlier sections. This approach is the relativistic analog of the method used in [5]. In the nonrelativistic case of [5] $\vec{\nabla}\theta$ is approximated by θ/R_0 where θ is given by (45). Expression (45) is a consequence of results obtained from solving nonrelativistic transport equations. In particular, the ratio of $\tilde{n}_v$ to θ, used to show that $\tilde{p}_v=0$, is found from nonrelativistic solutions of (33) and (34). The condition $\tilde{p}_v=0$ is used in (57) to obtain an explicit expression for θ at $r=R$. This same ratio must be obtained from the relativistic solutions to see if $\tilde{p}_v$ is still approximately zero. To do this we solve the relativistic equations for $\tilde{n}$ and θ using the expressions

$$\tilde{e}_v=A\theta+B\tilde{n}_v\,,$$

$$\tilde{p}_v=C^2\tilde{n}_v+\partial p/\partial T\,\theta. \tag{79}$$

The first and second equations of (74) may be combined to give

$$\kappa^2(A\theta+B\tilde{n})=C^2q^2\tilde{n}+\partial p/\partial T q^2\theta. \tag{80}$$

We have assumed that both $\tilde{n}$ and θ are given by the form $\exp[-q(r-R_0)]/r$. The third equation of (74) gives

$$0=\kappa\tilde{n}-\kappa(A\theta+B\tilde{n})-\lambda H\kappa^2\theta+\lambda Hq^2\theta+\frac{n}{w}\lambda Gq^2$$

$$\times(C^2\tilde{n}+\partial p/\partial T\theta)-\lambda G\kappa^2(C^2\tilde{n}+\partial p/\partial T\theta). \tag{81}$$

Here

$$H=-\frac{n}{w},$$

$$G=\frac{nT}{w^2}. \tag{82}$$

We drop κ^2 terms in (81) in comparison to κ terms. Thus we are led to solve for q_1 and q_2

$$\det\begin{vmatrix}\kappa^2B-C^2q^2 & A\kappa^2-(\partial p/\partial T)q^2\\ \kappa M+Nq^2 & X\kappa+Yq^2\end{vmatrix}=0.$$

Here

$$M=1-\frac{n}{w}B,$$

$$N=GC^2\lambda,$$

$$X=-\frac{n}{w}A,$$

$$Y=\lambda(H+G). \tag{83}$$

Two approximate solutions may be obtained, one with $q^2\propto\kappa^2$ and the other with $q^2\propto\kappa$:

$$q_1^2=\kappa\frac{-C^2X+M\partial p/\partial T}{C^2Y-N\partial p/\partial T},$$

$$q_2^2=\kappa^2\frac{X-MA}{\partial p/\partial TM}. \tag{84}$$

Substituting back into (80) one finds

$$\frac{\tilde{n}}{\theta}=\frac{\kappa^2B-c^2q_1^2}{A\kappa^2-(\partial p/\partial_T)q_1^2}, \tag{85}$$

where terms proportional to q_2^2 are dropped relative to q_1^2. Thus in the limit of small κ one obtains the same $\tilde{n}$ to θ ratio as in the nonrelativistic problem and one may thus use expression (45) for θ.

Combining the above information one finds

$$\left(-n_l l+\frac{\lambda 2\sigma T}{\kappa l n_l R_0^3}\right)\frac{w_v u}{\Delta w}=T\lambda\kappa u. \tag{86}$$

In the nonrelativistic limit the right-hand side of (86) disappears and one obtains the nonrelativistic result (59). One should note that (86) only holds in the small κ limit as this

approximation is used to obtain (45) and evaluate dT/dr. For the relativistic case, in the small κ limit, one obtains

$$\kappa = -\frac{w_v}{2T\lambda n_l l}\left[n_l l - \left(n_l^2 l^2 + \frac{8\lambda^2 T^2 \sigma}{R_0^3 w_v} \right)^{1/2} \right]. \tag{87}$$

By expanding the square root, assuming the λ^2 term is small, one recovers the nonrelativistic result of [1,5].

VI. RELATIVISTIC CASE: HEAT CONDUCTION AND VISCOSITY

To find an expression for κ in the presence of viscosity one may still use (78) because the third equation of (62) shows that viscous corrections to (78) are of second order in u. Viscosity, however, affects the expressions for $\vec{\nabla}\tilde{p}_v$ and θ written in terms of $\tilde{R}$. New expressions for these quantities must be found. To begin we find a relationship between $\tilde{n}_v$ and θ. To do so we define

$$\begin{aligned} \tilde{e}_v &\equiv S_a \tilde{n}_v + S_b \tilde{\theta} \\ &\equiv S_0 \tilde{n}_v \\ &\equiv S\theta. \end{aligned} \tag{88}$$

The coefficients S_a and S_b can be obtained from an expression for the energy density of the vapor material. Then it is possible to obtain

$$\frac{\tilde{n}_v}{\theta} = \frac{dp/dT}{C^2 F}, \tag{89}$$

which is almost identical to the nonrelativistic relation but with

$$F = 1 + \left(\frac{4}{3}\eta_v + \zeta_v \right) \frac{\kappa S_0}{w_v C^2} \tag{90}$$

instead of (64). The coefficients S and S_0 can be obtained given S_a and S_b together with the relationship (89). To begin we focus on $\vec{\nabla}\tilde{p}_v$ which may be expressed in terms of θ and u. To do this we note that in the presence of viscosity the second equation of (74) becomes

$$\partial_t w \vec{u} = -\vec{\nabla} p + \vec{\nabla}\left[\left(\zeta_v + \frac{4}{3}\eta_v \right) \vec{\nabla}\cdot\vec{\mathbf{u}} \right], \tag{91}$$

while the first and third equations remain unchanged to first order in u. One may use the first equation of (74) to obtain

$$\vec{\nabla} p = -w_v \kappa u - \left(\frac{4}{3}\eta_v + \zeta_v \right) \frac{\kappa}{w_v} S_3 \vec{\nabla}\theta. \tag{92}$$

The second term on the right-hand side of (92) is combined with $\lambda\vec{\nabla}T$ of (78) or

$$\begin{aligned} \lambda &\to \lambda \left[1 + \left(\frac{4}{3}\eta_v + \zeta_v \right) \frac{\kappa S_3 T}{w_v^2} \right] \\ &\equiv \lambda'. \end{aligned} \tag{93}$$

One may note that $\vec{\nabla}T = \vec{\nabla}\theta$. Next we obtain an expression for θ using (57). We use (65) and obtain $\tilde{p}_l$ from (17). One should note, however, that u is now the velocity of the energy flow and that one uses the first equation of (74) and the boundary conditions (75), with viscosity included (see Appendix B), instead of (4) and (6). As a result one finds

$$\theta = \frac{\tilde{R} E_1}{E_2}, \tag{94}$$

where

$$\begin{aligned} E_1(\kappa) = {} & \frac{-2\sigma}{R_0^2} + \frac{2\kappa}{R_0}\left(-\frac{4}{3}\eta_l - \zeta_l + \frac{4}{3}\eta_v + \zeta_v \right) \\ & + \kappa\left(\frac{4\eta_l}{R_0} - \frac{4\eta_v w_l}{w_v R_0} \right) \end{aligned} \tag{95}$$

and

$$E_2(\kappa) = \frac{-l n_l}{T} + \frac{\Delta n}{n_v}\left(1 - \frac{1}{F} \right)\left(\frac{\partial p}{\partial T} \right) - S_3 \frac{\kappa}{w_v}\left(\frac{4}{3}\eta_v + \zeta_v \right). \tag{96}$$

Finally the result for the relativistic κ, in the limit of small κ, in the presence of viscosity can be obtained. The same method used in the case with viscosity absent is applied. Now the equation to solve is

$$-T\lambda\kappa^2 - n_l l \kappa + \lambda' \frac{E_1}{E_2 R_0} = 0, \tag{97}$$

which yields

$$\kappa = \frac{-1}{2T\lambda}\left(n_l l - \sqrt{n_l^2 l^2 + \frac{4T\lambda\lambda' E_1}{E_2 R_0}} \right). \tag{98}$$

This expression for κ may be used in calculations of the nucleation rate of quark drops in superheated baryon material such as those of [3].

VII. DISCUSSION

We wish to make a numerical comparison of (3) and (98) for the case of quark-gluon drop formation in superheated baryon material. We use expressions and values for the necessary baryon and quark-gluon plasma properties given in the third reference of [3]. In [3] the nucleation rate, (1), is calculated using the expression (3) for κ. Expressions for Ω_0 and ΔF are given in [3]. We evaluate the various pieces of the rate using parameter set 1 from the third reference of [3] at an initial temperature and R_0 of 173 MeV and 0.4 fm, respectively. We find that $I \approx 2.6\times 10^{-6}$ c/fm^4 where $\Omega_0 = 0.006\,4$ $1/\mathrm{fm}^3$, $\exp[-\Delta F/T] = 0.056$, and $\kappa = 0.0074$ c/fm. To obtain the above we have used $\lambda = 0.3$ c/fm^2 and $\eta = 77.0$ MeV/fm^2 c and ignored ζ (see [3]). The important point to note about the evaluation of κ (3) is that the viscosity term $8/3\,\eta = 205$ MeV/fm^2 c is much larger than the term accounting for heat conduction, namely, $\lambda T = 51$ MeV c/fm^2. We assume that the speed of light, c, equals one. Thus it is the viscosity term that is responsible for most of the numerical value of κ. Since the nucleation rate I is mul-

tiplied by κ the viscosity has an important effect on the overall rate.

We now examine the results for κ (87) and (98) presented in this paper. We first note that the relativistic modification to the classical heat conduction term (72) changes the value of κ (87) by less than 1% from the classical result (59). This result is due to the property that $n_l l$, approximated here and in [3] by Δw, is very large compared to the second term in the square root of (87) for the superheated baryon matter problem. We nevertheless think that it is important to point out that the relativistic (87) and nonrelativistic (59) forms for κ are not the same as proposed in [4]. The main numerical importance of our expression for κ in comparison to (3), however, is the role of the viscosity terms. In the case of (98) it appears that the effect of viscosity is small because all of the terms in E_1 and E_2, containing viscosity, are multiplied by κ_0 (59) whose small value is 0.0015 c/fm. For example, we evaluate various pieces of E_1. The dominating term is proportional to $2\sigma/R_0 = 250$ MeV/fm^3 for $\sigma = 50$ MeV/fm^2. The viscosity terms, on the other hand, are $8\kappa\eta_l/3 \approx 8$ MeV/fm^3 for η_l, the viscosity of the quark-gluon material, equal to about 2000 MeV/fm^2 c [10] and $4\kappa w_l \eta_v / w_v \approx 2$ MeV/fm^3. Thus the small numerical value of κ strongly suppresses the contribution of the viscous terms. We do not evaluate E_2 here because values for C^2, S_0, and S are not readily available from tables given in [3] and must instead be obtained numerically from the equation of state [3]. We anticipate that the viscous terms in E_2 are also small because they are multiplied by κ. Thus using the expression for κ presented here, i.e., (98), in place of (3) reduces the nucleation rate by a factor of about 5. The conclusion to be drawn from the numerical results is that the viscous terms do not contribute appreciably to κ. This has important implications for the nonrelativistic results of Langer and Turski [1]. In the calculation of [1] the effects of viscosity are ignored even though a vapor with heat conduction is viscous [8]. Our results show that the viscosity terms are multiplied by κ in the nonrelativistic case (71). κ is small for the nonrelativistic case so viscosity terms can be ignored to good approximation in the expression of κ. Thus the Langer-Turski expression for κ, (59), is a good approximation even for systems with viscosity provided κ is small.

As a final note we wish to compare the scaling properties of (3) and (98). Kawasaki [7] has shown that (59) scales as $\epsilon^0 R^{-3}$, which is consistent with the requirement that κ scale as

$$\epsilon^3 f(\epsilon/R_0). \tag{99}$$

Here ϵ is the correlation length, i.e., the surface thickness of the drop, which tends to infinity as a negative power of $|T-T_c|$, and f is an arbitrary function. We note that λ scales as $\epsilon^{\gamma/\nu-1}$ while η scales as $|T-T_c|^0$ with $\epsilon \approx |T-T_c|^{-\nu}$ [11]. Here γ and ν are critical exponents described in [8]. We ignore ζ as it is small. Thus the scaling properties of (3) of [4] are dominated by the viscosity term which does not scale according to the requirement (99). For the expression for κ, (98), presented in this paper, the viscosity terms have been shown to be small because they are multiplied by κ. In addition, the nonrelativistic result (59) describes the relativistic situation for the superheated baryon material problem. Thus the numerical value and the scaling properties of κ are governed by those of (59), which scales properly.

VIII. CONCLUSION

We have examined the derivation of κ in the presence of heat conduction and viscous effects both for nonrelativistic and relativistic problems. We find that heat conduction and viscosity terms are not treated on the same footing as suggested by (3). Our expressions for κ with viscosity contain terms with viscosity coefficients multiplied by κ itself. If κ is small these modifications due to viscosity are also small. We also find that relativistic and nonrelativistic expressions for heat conduction differ in form. We are not able to make the simple connection with the nonzero baryon case [6] by setting λ equal to zero in our expression of κ as done in [4]. We present an expression for κ for a nonrelativistic medium with viscosity and finite heat conduction (71). We obtain an expression for κ for a relativistic, viscous vapor with finite heat conduction (98). Finally we show that the numerical difference in expressions (3) and (98) for κ reduces the nucleation rate for quark-gluon drops forming in superheated baryon material by roughly a factor of 5.

APPENDIX A

We evaluate here the factor A_2/A_1 in (43) using the expressions of [1] instead of those of Sec. V and show that in the limit of small κ the two results are the same. The computation of Ref. [1] differs from that of this paper in that transport equations are solved in an interface region between the pure liquid and vapor regions. The approach of this paper is to link the liquid and vapor regions with boundary conditions and treat the interface region as having zero thickness.

We solve for the ratio A_2/A_1 using the following expressions from [1]:

$$A_1 + A_2 = \tilde{a}(\infty) \Big/ \frac{\partial^2 f}{\partial n_v^2},$$

$$\frac{A_1}{(1-b)} + A_2 = \frac{n_v^2 c_v R\theta}{T\partial_T p}. \tag{A1}$$

Next we evaluate $\tilde{a}(\infty)$ using

$$\tilde{A} = \tilde{a}(\infty) + \frac{R\theta\partial_T p}{n_v} - \frac{R\theta n_l l}{T\Delta n} \tag{A2}$$

and

$$\frac{1}{3} n_l c_l \kappa\theta \approx \frac{-\kappa\tilde{A} l n_l \Delta n}{2\sigma} - \frac{\lambda\theta}{R^2}. \tag{A3}$$

We refer the reader to [1] for details concerning the origin of these equations. Note: l should be replaced with $l n_l/\Delta n$ in the expressions of [1] (see [5]). We thus obtain

$$\frac{A_2}{A_1} = \frac{T\partial_T p}{(1-b)bc^2 c_v n_v} \frac{\kappa c_l R_0^2 n_l^2 l}{\lambda T 3\Delta n}. \tag{A4}$$

An expression for c_l is needed to allow for comparison with (43). To obtain such a result use is made of

$$\frac{\kappa^2}{\kappa_0^2}=1-\alpha\frac{\kappa}{\kappa+\bar{\lambda}}, \tag{A5}$$

where

$$\alpha=\frac{3l^2n_lR_0}{2\sigma Tc_l},$$

$$\bar{\lambda}=\frac{3\lambda}{R_0^2n_lc_l}. \tag{A6}$$

Expressions for κ and κ_0 are given by (59) and (15) respectively. The small value limit of κ is obtained in [1] by dropping the left-hand side of (A5) and neglecting κ relative to λ. To obtain a result for c_l one retains the e left-hand side of (A5) and expands the right-hand side to first order in κ/λ and compares the coefficients of the κ^2 terms. This leads to

$$\frac{1}{\kappa_0^2}=\frac{\alpha}{\bar{\lambda}^2} \tag{A7}$$

and

$$c_l=\frac{(\Delta n)^2}{n_v}\frac{3Tm\lambda^2}{R_0^2l^2n_l^3} \tag{A8}$$

and ultimately to

$$\frac{A_2}{A_1}=m\kappa\lambda\frac{T\partial_T p\Delta n}{c_vn_v^2(1-b)n_llbc^2}. \tag{A9}$$

This is consistent with (43). Note, use has been made of the assumption of a small κ throughout both here and in [1].

In Sec. V condition (40), implying energy removal through pure heat conduction, is used. We now show that this condition is essentially included in the expression (A3). The second term on the right-hand side of (A3) is equivalent to $\lambda\vec{\nabla}\theta/R_0$ in the limit of small κ. The first term on the right-hand side is equivalent to $\Delta n\tilde{R}_0$. To see this one may consider the expression of [1]:

$$\frac{-R_0^3\tilde{A}(\Delta n)^2}{2\sigma}+A_2q_2^{-2}\approx 0 \tag{A10}$$

together with

$$4\pi\int r^2dr\tilde{n}\approx A_2q_2^{-2}4\pi, \tag{A11}$$

where the integral is taken over all space, the inside of the drop has been neglected, and terms involving q_1 in the vapor have been dropped. (It is interesting to note that near the drop surface

$$\tilde{n}\approx A_1/R_0 \tag{A12}$$

because $A_1\gg A_2$, but that over all space the approximation of [1] is to include only the A_2 part because it is multiplied by $\exp[-q_2(r-R_0)]$ [see (36)] and $q2\ll q1$ and so dies very slowly.) As a consequence the first term on the left-hand side of the first equation of (36) must represent the particle number change due to the drop growing from radius R_0 to $R_0+\tilde{R}$. This change is equivalent to

$$\Delta n\tilde{R}4\pi R_0^2. \tag{A13}$$

[One may note that $\dot{R}=\kappa\tilde{R}$. This allows for a connection between $\tilde{R}$ in (A13) and $\dot{R}$ in (40).] Thus the first term on the right-hand side of (A3) is

$$\frac{ln_l\kappa\tilde{R}}{R_0}. \tag{A14}$$

Finally one must consider the term on the left-hand side of (A3). This term involving c_l can be neglected in the limit of small κ used in [1]. It is also possible to calculate κ in the limit of small κ setting c_l to zero in (A3) and using an expression for $\tilde{A}$ derived from expressions in [1]. The result for κ is identical to (59). This shows that the c_l term in (A3) is small and can be neglected to first approximation. Thus (A3) with the c_l term dropped,

$$0\approx n_llk\tilde{R}+\lambda\frac{dT}{dr}, \tag{A15}$$

is equivalent to (40).

APPENDIX B

In this appendix we sketch a derivation of the boundary conditions (17), (18), (62), and (75) with viscosity added. For a relativistic system one may obtain boundary conditions, relating two different media, by equating the momentum-energy tensor

$$T^{ij}=pg^{ij}+(e+p)u^iu^j+\tau^{ij} \tag{B1}$$

in a frame moving with the interface between the two media [9]. Here u^i is the velocity of energy flow,

$$\tau^{ij}=-\eta\left(\frac{\partial u^i}{\partial x^k}+\frac{\partial u^k}{\partial x^i}+u^ku^l\frac{\partial u^i}{\partial x^l}+u^iu^l\frac{\partial u^k}{\partial x^l}\right)$$
$$-\left(\zeta-\frac{2}{3}\eta\right)\frac{\partial u^l}{\partial x^l}(g^{ik}+u^iu^k) \tag{B2}$$

and g^{ij} is the a diagonal metric with $g^{00}=-1$ and the rest of the diagonal elements equal to 1. The boundary condition involving pressure may be found equating $T^{ij}x^ix^j/r^2$ across the interface, i.e.,

$$T^{ij}\frac{x_jx_i}{r^2}=\left[p+\left(\frac{4}{3}\eta+\zeta\right)\vec{\nabla}\cdot(\vec{u}-\vec{R})+\frac{4\eta(u-\dot{R})}{r}\right] \tag{B3}$$

and adding in a term σ/R to account for the surface as done in [2,3]. (Note: For a relativistic system with no viscosity the condition

$$p_l-p_v=\frac{2\sigma}{R} \tag{B4}$$

is taken from [2,3].) Equating $T^{0i}x_i/r$ across the interface leads to the boundary condition

$$w_v(u_v-\dot{R})=w_l(u_l-\dot{R}). \tag{B5}$$

For the nonrelativistic case the pressure boundary conditon of (62) may be obtained by replacing u with the velocity of mass flow. For a discussion of boundary conditions for both the nonrelativistic and relativistic cases see [9].

[1] J. S. Langer and L. A. Turski, Phys. Rev. A **8**, 3230 (1973).
[2] L. P. Csernai and J. I. Kapusta, Phys. Rev. D **46**, 1379 (1992).
[3] L.P. Csernai and J. I. Kapusta, Phys. Rev. Lett. **69**, 737 (1992); M. Carrington and J. I. Kapusta, Phys. Rev. D **47**, 5304 (1993); J. I. Kapusta, A. P. Vischer, and R. Venugopalan, Phys. Rev. C **2**, 901 (1995).
[4] R. Venugopalan and A. Vischer, Phys. Rev. E **49**, 5849 (1994).
[5] L. A. Turski and J. S. Langer, Phys. Rev. A **22**, 2189 (1980).
[6] F. Ruggeri and W. Friedman, Phys. Rev. D **53**, 6543 (1996).
[7] K. Kawasaki, J. Stat. Phys. **12**, 365 (1975).
[8] K. Huang, *Statistical Mechanics* (John Wiley and Sons, New York, 1987).
[9] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, London, 1959).
[10] G. Baym, H. Monien, C. Pethick, and D. Ravenhall, Phys. Rev. Lett. **64**, 1867 (1990).
[11] L. Kadanoff and J. Swift, Phys. Rev. **166**, 89 (1968).